Team 25 Truss Design: Experimentation and Optimization

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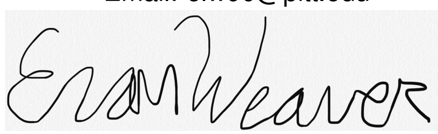

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Over the course of several weeks, our team designed several balsa wood trusses to support at least one hundred pounds and maximize the ratio of supported weight to truss weight times cost. Based on preliminary research of truss design methods and finite element analysis of several different trusses, our group elected to construct a Warren truss design, with zero force members to counteract bending and enough cross members to prevent failure. To verify that the bridge could support the prescribed load, our team conducted a 2D method of joints analysis on the truss, and found that stress values fell within the acceptable range for balsa wood. We constructed this design using provided fasteners, and we loaded it with ball-bearings. Our first design performed well and encountered cross member failure. After significant revisions and analyses similar to those described above, we tested our second truss and obtained a much higher index due a failure in the actual truss at a zero force member. During the final week of the project, we constructed our final design, very similar to our second truss that failed in the crossmembers due a fewer number of them. Through this experience, our team was able to evaluate the real-world performance of our designs after careful analysis and hands-on construction. We learned about the nuances of truss design, the importance of zero-force members and factors of safety, and the necessity to account for all failure modes in design.

Nomenclature

$[lbf]$ English unit of force
$[in]$ English unit of distance
$[psi]$ English unit of stress, [lbf/in^2]
$[lb]$ English unit of mass
$[-]$ Indicates a unitless measure
$\circ$ Unit of angular measure
A Indicates a point and/or joint on a free body diagram
AB The force existing in and directed along the member between two joints, in this case joints A and B, measured in [lbf]
AB_x X-component of the force existing in a member, in this case member AB, in [lbf]
AB_y Y-component of the force existing in a member, in this case member AB, in [lbf]
M_A Moments existing about a joint (joint A), in [lb $\cdot in$]
△ Pin support (provides x- and y- reactionary forces)
△ Roller support (provides y- reactionary forces only)
σ_n Maximum normal stress existing in a member, in [psi]
τ_s Maximum shearing stress existing in a member, in [psi]
σ_b Bearing stress existing in a member, in [psi]

θ Angle existing between the horizontal and member AB (exists throughout the truss due to the Converse of the Parallel Lines Theorem), in °

$A_{\perp}$ Cross sectional area of a member with respect to a plane perpendicular to the axial dimension, in [in^2]

FS Factor of safety [-]

↑ Directional arrow indicating the direction of the associated force

The application of these variables to the actual truss shall be explored within the methodology section.

1 Introduction

Trusses have historically been a means of designing very strong structures while simultaneously saving weight and cost [1]. The advantage of using trusses to support loads comes from the fact that they are able to distribute uneven loads relatively evenly across their length. They are used in many structural applications today, from bridges to buildings to automobile chassis. As human knowledge of engineering concepts and material properties has advanced, so too has truss design. One helpful tool is the construction of a scale model truss using balsa wood, and is frequently used in educational settings to help students gain an understanding of the real-life applications of engineering. The following section will discuss a brief history of truss design and present some considerations when designing a balsa wood truss.

Truss designs existed before the 18th and 19th centuries but they were severely limited. We have seen evidence of trusses being used to reinforce structures such as Egyptian boats and in several bridge spans in 16th- and 17th- century Germany [1]. However, most of the engineering behind these structures occurred through trial and error. A general understanding of forces and stress (i.e. knowledge of their presence) existed during these times, but rational design did not occur. It was not until the 1820s when Claude-Louis Navier published his *Résumé des Leçons Données a L'École des Ponts et Chaussées sur L'Application de la Mécanique* that trusses could be mathematically analyzed [1]. His work and the concepts presented to engineers at the School laid the groundwork for modern truss design; Camille Polonceau, a student at another French institution, created the Polonceau truss in 1837, a widely used roof trusses during this time [1].

Fig. 1 Polonceau Roof Truss

Truss innovation was not limited to France, however. In Britain, Robert Stephenson worked with iron to design trusses with applications in the railway industry [1]. One such example of his work exists at the Chalk Farm Locomotive roundhouse in Birmingham [1]. He and William Doyne proposed alternative designs for the chords of the truss sections, with Doyne proposing a lattice design and Stephenson proposing a solid girder [1]. James Warren in 1848 proposed a design that included alternating tension and compression members [1]. While this design was more complex to construct, it made more efficient use of material resources than alternative designs at the time.

Fig. 2 Stephenson's Roundhouse Roof Truss

The United States, however, saw the greatest extent of truss design implementation due to its abundant timber resources and increasing innovation in the iron and steel production industries [1, 2]. Theodore Burr, Thomas Palmer, and Ithiel Town were among the chief names in truss design during the 1820s and 1830s [1]. Town invented a truss, referred to as the "lattice", that consisted of criss-crossing diagonal members, and the design was heavily used during this time [3]. Palmer was influential in the design of covered bridges, which contributed greatly to the weatherproofing of the wooden structures. Stephen Harriman Long proposed the Long truss in 1830, which made improvements to the design in Navier's *Leçons* [1]. He applied the concepts of prestressing to his designs, which eliminated the need for tension bracings in diagonal members. Perhaps the two most famous American designs came in 1840 and 1844: respectively the Howe and Pratt trusses [3]. The Howe had diagonal compression members made of wood and vertical tension members made of iron; the Pratt had diagonal tension members made of iron and vertical compression members made of wood [3]. The former was more prevalent during this era because the Pratt required more iron, which was still relatively expensive at the time. However, as the cost of iron fell, the designs became more equally used for railroad bridge applications.

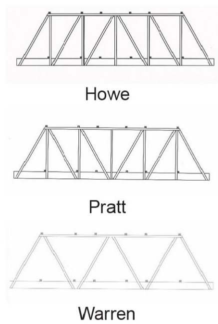


Fig. 3 Popular Truss Designs: Howe, Pratt, Warren

Fig. 4 Pennsylvania Truss, a Variation on the Pratt that Includes Arches

Today, trusses take a variety of forms, although we still use many of the concepts set forth by these early examples of truss design. One important tool that engineers use to simulate the behavior of bridges is the construction of a balsa wood truss. In order for the design to be successful, several factors must be considered. Balsa wood is stronger in tension than in compression [4], but with tension members one must also consider the narrowing of the cross section at the bearings. Additionally, while saving weight is important, trusses must also protect against cross member failure and bending. This can be accomplished by adding sufficient cross members to the top and vertical zero force members reduce bending, and distributing enough cross members on the bottom to reduce the shear experienced by an individual cross member. Zero force members help to prevent bending of the main beams and should therefore be included in design due to the minimal weight that they add.

Taking into consideration the rationale behind these methods of truss design, we have selected several layouts for our balsa wood bridge that will account for the limitations of balsa wood described above while attempting to maximize supported weight per bridge weight times bridge cost. The development and evaluation of these designs will be explored in subsequent sections.

2 Methodology

In order to ensure that our truss could withstand the prescribed loading conditions, we conducted a method of joints analysis on a 2D model of our truss coupled with a “worst-case” stress analysis for the balsa wood. An in-depth analysis of our first truss will be presented, and the same methodology will be used for further analysis of subsequent designs.

Before solving for forces existing in each member, the reactionary forces existing at the theoretical pin and roller supports were determined by constructing a global free-body diagram:

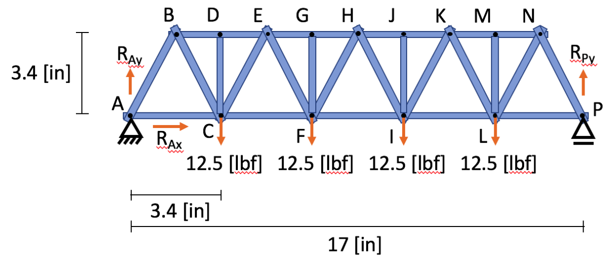


Fig. 5 Global Free-Body Diagram

Note that the load is approximated as a distributed 50 [lbf] load distributed evenly across the bottom joints of the truss. In order for the truss to be in equilibrium, the sum of the forces acting on the truss in the x- and y- directions and about any point must each sum to zero. For this analysis, the moments existing about point A, denoted by M_A, will be considered. Noting the applied and reactionary forces, denoted by ”R”, on the truss, the three unknown reactionary forces can be solved as follows:

$$\Sigma F_x = 0[lbf]$$

$$\boxed{R_{Ax} = 0[lbf]}$$

$$\Sigma M_A = 0[lb \cdot in]$$

$$(-12.5[lbf])(3.4[in]) + (-12.5[lbf])(6.8[in]) +$$

$$(-12.5[lbf])(10.2[in]) + (-12.5[lbf])(13.6[in]) +$$

$$R_{Py}(17[in]) = 0[lb \cdot in]$$

$$\boxed{R_{Py} = 25[lbf] \uparrow}$$

$$\Sigma F_y = 0[lbf]$$

$$-4(12.5[lbf]) + 25[lbf + R_{Ay} = 0[lbf]$$

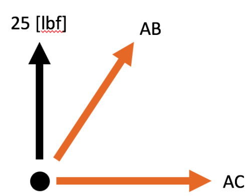
$$\boxed{R_{Ay} = 25[lbf] \uparrow}$$

Also note the angle value that exists at all diagonal members in the truss:

$$\theta = arctan(\frac{3.4[in]}{1.7[in]}) = 60^\circ$$

For the truss to be in equilibrium, the sum of forces in the x- and y- directions existing in the members at the joints must sum to zero. Additionally, because our trusses are symmetric, force values existing in half of the truss can be mirrored across an axis through the midpoint of the truss (joint H for our first design). Note that all members are initially assumed to be in tension, and a negative force value implies compression. Additionally, orange forces are unknown for that particular joint. The sum of forces in the x- and y- directions at each joint are solved for joints A-G in the following calculations, moving left to right across the truss. By solving the equations as such, each joint is statically determinate. Note that "AB" represents the force in member AB, and the previously calculated angle value of 60°exists throughout the truss:

Joint A



$$\Sigma F_y = 0[lbf]$$

$$25[lbf] + AB_y = 0[lbf]$$

$$25[lbf] + ABsin(60^\circ) = 0[lbf]$$

$$\boxed{AB = -28.868[lbf]}$$

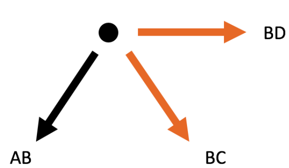
$$\Sigma F_x = 0[lbf]$$

$$AB_x + AC = 0[lbf]$$

$$ABcos(60^\circ) + AC = 0[lbf]$$

$$\boxed{AC = 14.434[lbf]}$$

Joint B



$$\Sigma F_y = 0[lbf]$$

$$-AB_y - BC_y = 0[lbf]$$

$$-ABsin(60^\circ) - BCsin(60^\circ) = 0[lbf]$$

$$\boxed{BC = 28.868[lbf]}$$

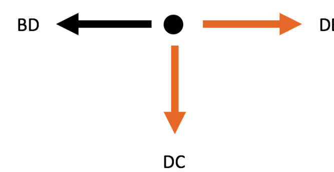
$$\Sigma F_x = 0[lbf]$$

$$-AB_x + BC_x + BD = 0[lbf]$$

$$-ABcos(60^\circ) + BCcos(60^\circ) + BD = 0[lbf]$$

$$\boxed{BD = -28.868[lbf]}$$

Joint D



$$\Sigma F_y = 0[lbf]$$

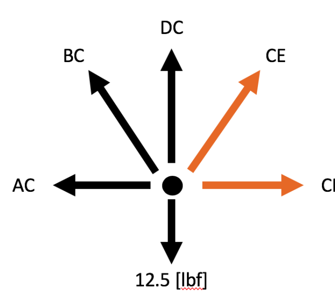
$$\boxed{DC = 0[lbf]}$$

$$\Sigma F_x = 0[lbf]$$

$$-BD + DE = 0[lbf]$$

$$\boxed{DE = BD = -28.868[lbf]}$$

Joint C



$$\Sigma F_y = 0[lbf]$$

$$BC_y + CE_y + DC - 12.5[lbf] = 0[lbf]$$

$$BCsin(60^\circ) + CEsin(60^\circ) + DC - 12.5[lbf] = 0[lbf]$$

$$\boxed{CE = -14.434[lbf]}$$

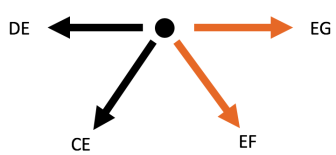
$$\Sigma F_x = 0[lbf]$$

$$-AC - BC_x + CE_x + CF = 0[lbf]$$

$$-AC - BCcos(60^\circ) + CEcos(60^\circ) + CF = 0[lbf]$$

$$\boxed{CF = 36.085[lbf]}$$

Joint E



$$\Sigma F_y = 0[lbf]$$

$$-CE_y - EF_y = 0[lbf]$$

$$-CEsin(60^\circ) - EFsin(60^\circ) = 0[lbf]$$

$$\boxed{EF = 14.434[lbf]}$$

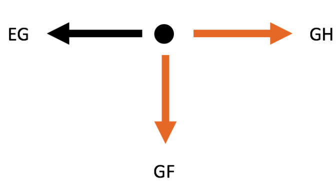
$$\Sigma F_x = 0[lbf]$$

$$-DE - CE_x + EF_x + EG = 0[lbf]$$

$$-DE - CEcos(60^\circ) + EFcos(60^\circ) + EG = 0[lbf]$$

$$\boxed{EG = -43.301[lbf]}$$

Joint G



$$\Sigma F_y = 0[lbf]$$

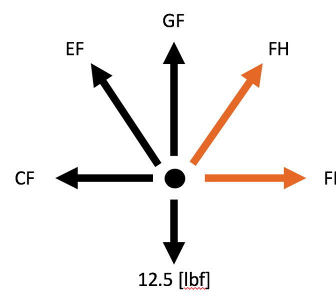
$$\boxed{GF = 0[lbf]}$$

$$\Sigma F_x = 0[lbf]$$

$$-EG + GH = 0[lbf]$$

$$\boxed{GH = EG = -43.301[lbf]}$$

Joint F



$$\Sigma F_y = 0[lbf]$$

$$EF_y + FH_y + GF - 12.5[lbf] = 0[lbf]$$

$$EFsin(60^\circ) + FHsin(60^\circ) + GF - 12.5[lbf] = 0[lbf]$$

$$\boxed{FH = 0[lbf]}$$

$$\Sigma F_x = 0[lbf]$$

$$-CF - EF_x + FH_x + FI = 0[lbf]$$

$$-CF - EFcos(60^\circ) + FHcos(60^\circ) + FI = 0[lbf]$$

$$\boxed{FI = 43.301[lbf]}$$

Noting the symmetry of the truss, the forces in the members mirror about an axis through joint H. The following table summarizes the above force data and relates forces across the axis of symmetry:

Force	Magnitude [lbf]	State
AB=NP	28.868	Compression
AC=LP	14.434	Tension
BC=LN	28.868	Tension
BD=MN	28.868	Compression
DC=ML	0	-
DE=KM	28.868	Compression
CE=KL	14.434	Compression
CF=IL	36.085	Tension
EF=IK	14.434	Tension
EG=JK	43.301	Compression
GF=JI	0	-
GH=HJ	43.301	Compression
FH=HI	0	-
FI	43.301	Tension

These calculations were performed in Microsoft Excel using an augmented matrix to solve. We then obtained the highest force values for tension and compression in a member and performed a "worst-case" analysis. Note that the dimensions of the bottom beam are 3/4[in] x 3/8[in], and other members are 3/8[in] x 3/8[in]. The highest tensile force occurred in member FI, but because this member has a greater cross section we also considered the highest tensile force in a member with a smaller cross section. For this truss, this could be any tensile diagonal member. The highest compressive stress occurred in members EG, GH, HJ, and JK, which already had a smaller cross section. The calculations are as follows:

Member FI

$$\sigma_n = \frac{FI}{A_\perp}$$

$$\sigma_n = \frac{43.301[lbf]}{(0.75-0.164)(0.375)[in^2]}$$

$$\boxed{\sigma_n = 197.047[psi]}$$

Note that the area of the bearing is subtracted from the cross sectional area of tensile members. This occurs because the narrow cross section at the bearing experiences the full tensile force in the member and is thus the weakest point. Maximum normal and shear stresses will occur here. Maximum shear stress is half of the maximum normal stress:

$$\tau_s = \frac{FI}{2A_\perp}$$

$$\tau_s = \frac{43.301[lbf]}{2(0.75-0.164)(0.375)[in^2]}$$

$$\boxed{\tau_s = 98.523[psi]}$$

Bearing stress is a compressive force that acts on the projection of its cross sectional area onto the member, hence the negative value for the stress. Bearing stress for member FI is calculated as follows:

$$\sigma_b = \frac{FI}{A_b}$$

$$\sigma_b = \frac{43.301[lbf]}{(0.164)(0.375)[in^2]}$$

$$\boxed{\sigma_b = (-)704.081[psi]}$$

These calculations were repeated for the maximum tensile force in a member other than the larger bottom ones to verify that the bottom is in fact the "worst case". This calculation pertains to any diagonal tension member:

$$\sigma_n = \frac{BC}{A_\perp}$$

$$\sigma_n = \frac{28.868[lbf]}{(0.375-0.164)(0.375)[in^2]}$$

$$\boxed{\sigma_n = 364.840[psi]}$$

$$\tau_s = \frac{BC}{2A_\perp}$$

$$\tau_s = \frac{28.868[lbf]}{2(0.375-0.164)(0.375)[in^2]}$$

$$\boxed{\tau_s = 182.420[psi]}$$

$$\sigma_b = \frac{BC}{A_b}$$

$$\sigma_b = \frac{28.868[lbf]}{(0.164)(0.375)[in^2]}$$

$$\boxed{\sigma_b = (-)469.398[psi]}$$

Now we will consider the maximum compressive force, which already exists in a member with smaller cross sectional area. Note that the area of the bearing is not removed for compressive members because the narrow cross section does not experience the force existing between the bearings. The stress analysis for members EG, GH, HJ, and JK are as follows:

$$\sigma_n = \frac{EG}{A_\perp}$$

$$\sigma_n = \frac{-43.301[lbf]}{(0.375)(0.375)[in^2]}$$

$$\boxed{\sigma_n = -307.918[psi]}$$

$$\tau_s = \frac{EG}{2A_\perp}$$

$$\tau_s = \frac{-57.736[lbf]}{2(0.375)(0.375)[in^2]}$$

$$\boxed{\tau_s = -153.959[psi]}$$

$$\sigma_b = \frac{EG}{A_b}$$

$$\sigma_b = \frac{-43.301[lbf]}{(0.164)(0.375)[in^2]}$$

$$\boxed{\sigma_b = -704.081[psi]}$$

The tensile strength of medium-density balsa wood is 2886.251 [psi], the compressive strength is 1754.957 [psi], and the shear strength is 565.6 [psi] [4]. Bearing stress in member FI is the highest stress that occurs in the truss, and it is compressive. Thus, the minimum factor of safety can be obtained from this value:

$$FS = \frac{\sigma_{cs}}{\sigma_{max}}$$

$$FS = \frac{1754.957[psi]}{704.081[psi]}$$

$$\boxed{FS = 2.493[-]}$$

Our factor of safety is comfortably above 1 for this design, and so we proceeded to build and test this design. Based on the results described in the next section, we made several revisions to our design in an effort to reduce weight and cost of our truss. We moved to a two-triangle Warren truss with zero-force members:

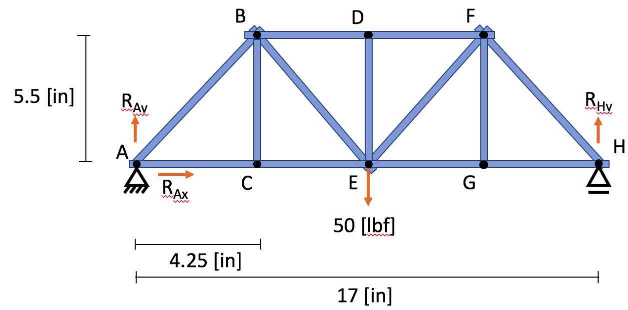


Fig. 6 Global Free-Body Diagram For Second Truss

We repeated the method described above for this truss, except that we treated the applied load as a point load to obtain a more conservative estimate of the performance of our bridge. The point load is less effectively distributed throughout the truss, thus creating higher forces in individual members. The force values, maximum tensile, compressive, and bearing stresses stresses, and factor of safety are summarized in the table below:

Force	Magnitude [lbf]	State
AB=FH	31.954	Compression
AC=GH	19.318	Tension
BC=FG	0	-
BD=DF	39.076	Compression
CE=EG	19.318	Tension
BE=EF	31.954	Tension
DE	0	-

Maximum Tensile Stress	403.842 [psi]
Maximum Compressive Stress	277.874 [psi]
Maximum Shearing Stress Stress	201.921 [psi]
Maximum Bearing Stress	635.382 [psi]
Factor of Safety	2.762 [-]

Once again, these calculations were performed in Microsoft Excel using an augmented matrix. We decided to construct this truss because the factor of safety was comfortably above 1. Based on the performance of this design, we elected to use the same truss design for our final bridge with modifications only to cross members and not to the existing truss structure. In all cases, cross members were added as deemed sufficient to support the loading plate.

3 Results and Discussion

To begin, our first design idea was to use a Warren truss. We conducted finite element analysis of a basic Warren truss without zero force members in ANSYS. The simulation showed significant bending that occurred under the prescribed loading conditions, which led us to incorporate zero force members into our first design to counteract the bending. Based off this design, we used a two dimensional force analysis, which was outlined in the methodology section, that showed that the bridge should be able to hold one hundred pounds with a factor of safety of about 2.5. These models made on paper or through simulation were helpful for understanding how the bridge would handle loads within the elastic region of the balsa wood, but when the load was outside of the elastic region and closer to the failure point of the balsa, it was impossible to predict how exactly the wood would act. This is where the physical testing became helpful.

The physical testing of our bridges yielded interesting results that we did not foresee in our original analysis. Our first bridge design had a more robust and strong truss structure to guarantee that we could hold the prescribed weight. We used 7 cross members on the bottom to connect the two trusses, and three on the top to prevent bending. Our bridge failed when the bottom cross members all simultaneously ruptured at 255.6 [lbf]. This showed us that our truss design was slightly over-designed, and more consideration should be given to cross member failure. This caused us to test the weight held by one piece of 5 [in] balsa wood and extrapolate how many members we would need for our bridge to fail at our point of max stress rather than at the cross members.

Fig. 7 Ball Bearings Transferred to Empty Bucket Suspended from Bridge for Testing

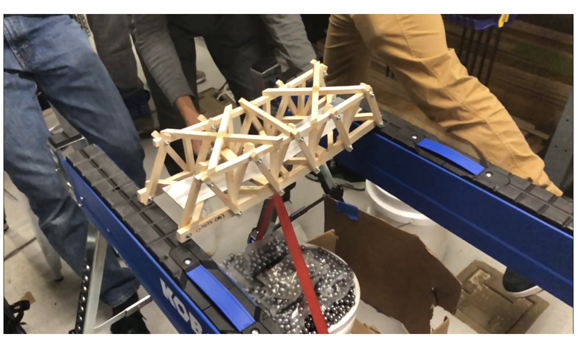


Fig. 8 Testing our First Bridge. Taken Before Cross Member Failure.

A detailed cost analysis of our bridge, along with its weight and performance ratios, are presented in the following table:

Component	length [in]	Unit Price	Quantity Used	Cost [dollars]
1" 8/32 Bolts	-	$0.09/bolt	4	$0.36
1 1/2" 8/32 Bolts	-	$0.14/bolt	18	$2.52
8/32 Nuts	-	$0.03/nut	22	$0.66
No. 8 Washers	-	$0.04/washer	22	$0.88
Long Bottom Beam	18.00 [in]	$0.05/inch	2	$1.80
Long Top Beam	14.60 [in]	$0.04/inch	2	$1.17
Diagonal Member	4.30 [in]	$0.04/inch	20	$3.44
Vertical Member	2.84 [in]	$0.04/inch	8	$0.91
Cross Member (X-shape)	6.37 [in]	$0.04/inch	10	$2.55
Cross Member (straight)	5.75 [in]	$0.04/inch	4	$0.92

Bridge Weight	0.5235 [lb]
Supported Weight	255.6 [lbf]
Bridge Cost	$15.21
Force/Weight Ratio	488.3 [lbf/lb]
Force/(Weight*Cost) Ratio	32.1 [lbf/($lb)]

Having met the requirement of holding 100 [lbf] total, our group focused on optimizing the force per weight-cost ratio. Given that the trusses were excessively strong in the first iteration, and that reducing the amount of material used will decrease cost and weight, we simplified our design by reducing truss members, which significantly reduced the total material. Although the simpler design was expected to be weaker due to less overall stiffness, the large reductions in weight and cost more than made up for it. A minimalist truss design allowed us to incorporate sufficient cross members in the design to prevent cross-member failure and force the actual truss to fail.

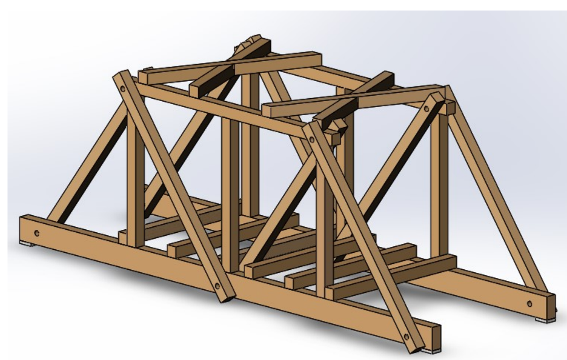


Fig. 9 Dimetric View of Bridge CAD in SolidWorks

This updated, simpler design proved to be very successful. The second bridge was significantly lighter and cheaper than the first design, and held 242.2 [lbf], which is only 13.4 [lbf] less than the first design. At the time, it produced a class record for the force per weight ratio. For the force per weight-cost ratio, which the bridge's final success will be measured by, the second bridge achieved double what the first bridge did at 64.3 [-] (versus 32.1[-] on the first bridge). The bridge failed when the wood glue on the middle zero force member on one side of the truss failed, causing the rest of the truss to rupture almost instantaneously. A detailed cost analysis of our second bridge, along with its weight and performance ratios, are presented in the following table:

Component	length [in]	Unit Price	Quantity Used	Cost [dollars]
1" 8/32 Bolts	-	$0.09/bolt	4	$0.36
1 1/2" 8/32 Bolts	-	$0.14/bolt	6	$0.84
8/32 Nuts	-	$0.03/nut	10	$0.30
No. 8 Washers	-	$0.04/washer	5	$0.20
Long Bottom Beam	18.00 [in]	$0.05/inch	2	$1.80
Long Top Beam	9.25 [in]	$0.04/inch	2	$0.74
Diagonal Member	7.88 [in]	$0.04/inch	8	$2.53
Vertical Member	4.94 [in]	$0.04/inch	6	$1.19
Cross Member (X-shape)	6.22 [in]	$0.04/inch	4	$1.00
Cross Member (straight)	5.75 [in]	$0.04/inch	7	$1.61

Bridge Weight	0.3565 [lb]
Supported Weight	242.2 [lbf]
Bridge Cost	$10.57
Force/Weight Ratio	679.4 [lbf/lb]
Force/(Weight*Cost) Ratio	64.3 [lbf/($lb)]

With the second bridge being ranked number one on the unofficial class leader board, the group decided to use the second design again for our final testing. We feared making major changes to the design because if the changes made the bridge fail earlier, we would be stuck with that result. When building our final bridge for test day, we paid careful attention to the length of zero-force members to ensure that the wood glue could set properly, and minor changes were made to the fabrication and cross member design to further lower the mass of the bridge while keeping its strength. We removed all washers, cut off the ends of the bolts, and removed material from the ends of compression members all to reduce our final weight and cost of the bridge. Additionally, we removed one cross member to save weight and cost, and we shifted the position of the remaining six to be more symmetric. Our final bridge on the official test day held 226.2 [lbf] and weighed 0.2955 [lb]. A detailed cost analysis of our final bridge, along with its weight and performance ratios, are presented in the following table:

Component	length [in]	Unit Price	Quantity Used	Cost [dollars]
1" 8/32 Bolts	-	$0.09/bolt	4	$0.36
1 1/2" 8/32 Bolts	-	$0.14/bolt	6	$0.84
8/32 Nuts	-	$0.03/nut	10	$0.30
Long Bottom Beam	18.00 [in]	$0.05/inch	2	$1.80
Long Top Beam	9.25 [in]	$0.04/inch	2	$0.74
Diagonal Compression Member	7.62 [in]	$0.04/inch	4	$1.22
Diagonal Tension Member	7.87 [in]	$0.04/inch	4	$1.26
Vertical Member	5.13 [in]	$0.04/inch	6	$1.23
Cross Member (X-shape)	5.87 [in]	$0.04/inch	4	$0.94
Cross Member (straight)	5.37 [in]	$0.04/inch	6	$1.29

Bridge Weight	0.2955 [lb]
Supported Weight	226.2 [lbf]
Bridge Cost	$9.98
Force/Weight Ratio	765.5 [lbf/lb]
Force/(Weight*Cost) Ratio	76.7 [lbf/($lb)]

4 Conclusions

From our research, design, and testing, we were able to reach several conclusions about our project. Our first design was aimed at meeting the required one hundred pound load minimum. It surpassed the requirement and held 255[lbf], but achieved a low performance index. We found here that many cross members must be used to hold high weights. For the second iteration, we aimed to decrease the weight and cost of our truss to improve the performance index. The second bridge used much less material, and still held a large amount of weight. This update doubled our performance index.

We learned here that the decrease in supported weight was less detrimental than previously thought, and that our priority was to aim for a low-cost, low-weight design to maximize performance. Another important takeaway from comparing the first and second bridge testing was the inaccuracy of the 2D truss analysis and the factor of safety calculation. Our second design had a higher factor of safety compared to the first design, however it held less weight. We knew the higher factor of safety was probably unrealistic because the second design had much less material to prevent bending failure. While the addition of zero force members help to prevent bending, we also realized a fabrication flaw that reduced the zero force members' effectiveness. We believe that bending in the truss, for which the 2D analysis does not account, also needs to be considered to get a realistic factor of safety. We moved forward with our final truss design that was, essentially, a replica of the second. In our last test, we were able to support 226.2 [lbf], and had a final performance index of 76.7 [lbf/($*lb]). The failure mode once again was cross member failure, but we deemed the weight and cost savings substantial enough to reduce the number of cross-members from our second design. Overall, through our design process we saw the importance of accounting for bending in our designs, and the value of a simple solution over a complex one.

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