Real World Dynamics: 2011 Nissan Altima

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Description:

Perhaps not one singular invention has revolutionized life to the extent that the automobile has. It allows for trips that once took weeks to occur in a matter of hours. This industry has always been of great interest to me. After taking rigid body dynamics and my other physics classes in college, I am in a position to understand much of what goes on in a vehicle and apply what I've learned to these systems. I have always been curious about how cars work, and I believe that using data obtained from my own vehicle, a 2011 Nissan Altima I can learn some new things about it. Rigid body dynamics is integral to the design and operation of such machinery. The pressure from the combustion of the fuel causes a force on the pistons, which rotates the crankshaft (rotational motion & relative velocity). The crankshaft drives the gears in the transmission (more rotational motion, gear ratios), which turn the driveshaft that is connected to the final drive system. The final drive system transmits the torque to the drive axle(s) (torque & rotation), which drive the wheels and move the car (kinetics of rigid bodies). The aerodynamic and frictional forces that oppose the motion of the vehicle, along with the power output, limit the vehicle's performance (equilibrium, systems of particles). Using data obtained from the internet, experimental data, and knowledge of rigid body dynamics, we can solve complex problems and learn how what we do applies in real life, and I'll learn more about my car than I knew before.

Relevant Data:

In order to solve the following example problem, I've obtained some pertinent data that will contribute to the experiment. The following table summarizes the data that is relevant to the calculations required for the example problem:

SAE Net Horsepower [1]	175 [hp]
SAE Net Torque [1]	180 [ft*lb]
First Gear Ratio [2]	3.50:1
Second Gear Ratio [2]	1.95:1
Third Gear Ratio [2]	1.39:1
Fourth Gear Ratio [2]	1.06:1
Fifth Gear Ratio [2]	0.81:1
Sixth Gear Ratio [2]	0.67:1
Final Drive Axle Ratio [2]	3.81:1
Wheel & Tire Diameter	26 [in]
Weight (including driver) [2]	3350 [lb]
Drag Coefficient [3]	0.31 [-]
Cross-Sectional Frontal Area [3]	23.0 [ft^2]
C-C Distance of Connecting Rods [4]	6.495 [in]
Rolling Friction Coefficient [5]	0.02 [-]
Redline	6250 [rpm]
Density of Air	2.38 E-3 [slugs/ft^3]

Example Problem:

Given the data in the previous section, help Frank to learn a little more about his 2011 Altima.

1.) What is the theoretical top speed of the Altima?
    a. Assume 85% efficient energy transfer from the engine to the wheels.
    b. Be sure to consider resistive forces (air resistance at top speed and friction) as well as the limits of the transmission.
    c. You may use MATLAB to solve any third order polynomials (hint).

2.) What is the theoretical time for the Altima to reach 60 mph from rest?
    a. Assume 85% efficient transmission of SAE Net Torque to the wheels.
    b. Assume no slippage of the wheels occurs during acceleration.
    c. The Nissan Altima uses a CVT, which has very smooth, efficient shifts. Therefore, neglect the time intervals where the car is "shifting" gears.
    d. Do consider resistive forces in some capacity. Hint: rolling friction can be treated as a constant value (treated as an external force not necessarily creating a torque at the wheels) and you can calculate the average value of the air resistance function from 0-60 mph.
    e. Note that wheel torque is a function of linear velocity. Use the average value of the torque function over the velocity interval for each gear in your calculations. Note that you must multiply this average value by the efficiency given in a.) Using the torque curve found at this link, I am providing you with a best fit curve that fits the data with correlation coefficient > 0.98 [-] in both cases:

$$T_1(V) = -1.74494V^2 + 118.20906V + 420.30194\ [ft \cdot lb]$$
$$T_2(V) = -0.30160V^2 + 36.69306V + 234.16823\ [ft \cdot lb]$$

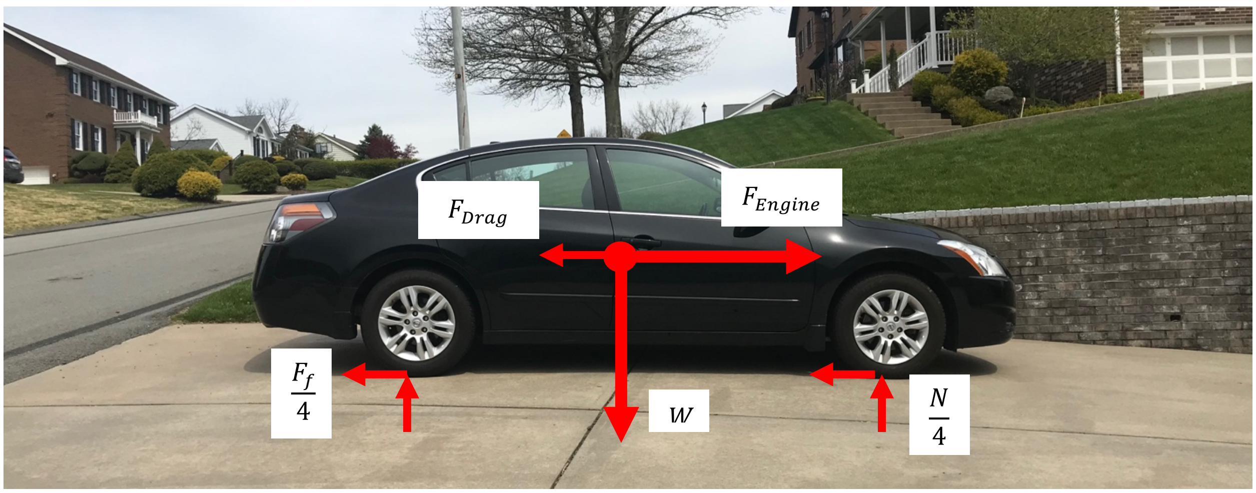
3.) What is the magnitude of the linear velocity of the pistons in the cylinder at the redline when the piston is halfway between top dead center and bottom dead center positions? What is the acceleration experienced by the piston at the redline at top dead center, given the rate of acceleration in first gear from Part 2.)?
    a. Assume that the radius, center-to-center, of the crankshaft is 2" and the length of the connecting rod, center-to-center, is 6.495" as stated in the table.
    b. Hint: Assume the engine goes from 0 [rpm] to the redline in the time interval for first gear you calculated in 2.), even though this is not the case in real life.
    c. Hint: The engine is an I4, meaning that the pistons travel vertically directly above the crankshaft.

Note: Due to the infeasibility of opening up the hood and manually measuring the characteristics of the engine like gear ratios and connecting rod dimensions, the only measurements I manually took were the wheel and tire diameter and redline (by inspection). The numbers next to a measurement in the table contain a link to their source.

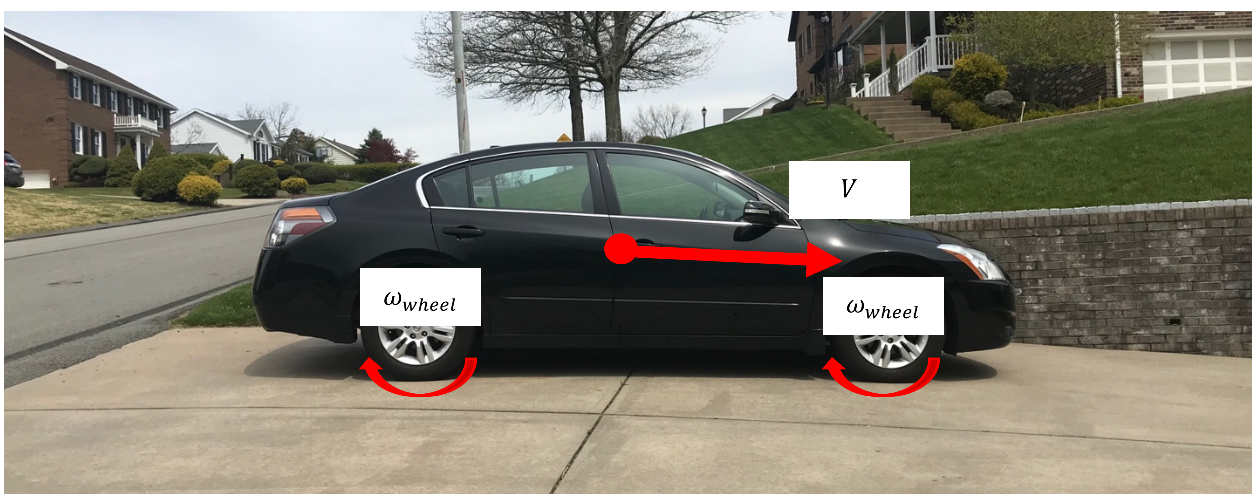
Solution

Part 1.) Calculate the theoretical top speed of the Altima.

This analysis will consider several factors that limit the top speed of a vehicle. By making several simplifications, we can assume that at the vehicle's top speed, the forces propelling the car forward are in equilibrium with the forces opposing the motion of the vehicle, mainly in the form of air resistance and rolling friction. Let us first consider a free-body diagram of the vehicle:



The kinetic diagram contains linear velocity of the vehicle and rotational velocity of the wheels, but no net acceleration:



We also know that:

$$P = \overrightarrow{F_{net}} \cdot \vec{V}$$

We know the power of the vehicle and thus can create an equation to solve for the theoretical maximum velocity of the Altima.

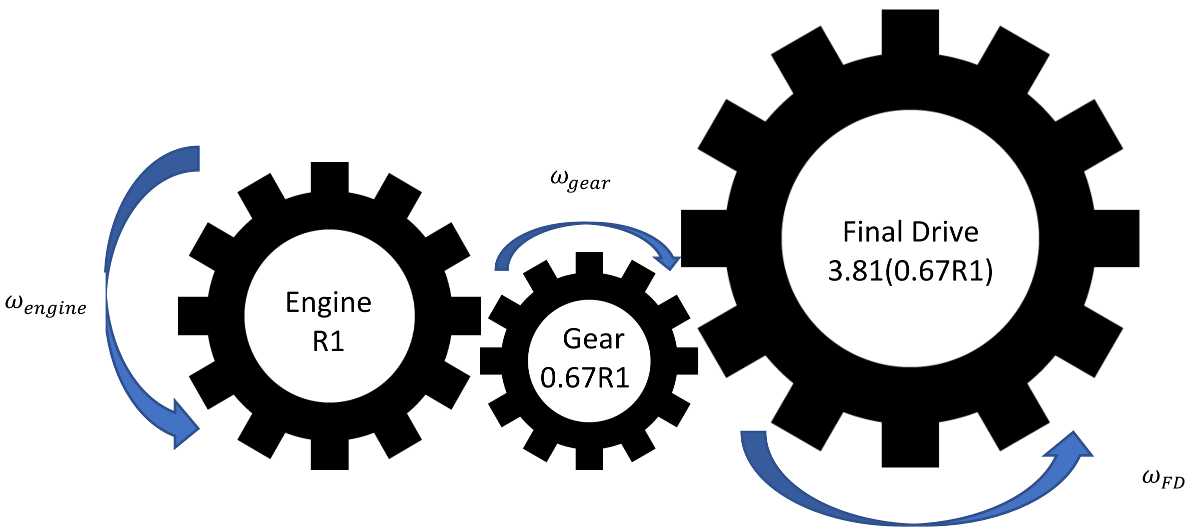
$$P_{net} = \left(\frac{1}{2} c_d \rho A V^2 + \mu_r F_n\right) V$$

$$(0.85\,[-])(175\,[hp])\left(550\,\frac{ft \cdot lb}{s \cdot hp}\right) = \frac{1}{2}(0.31\,[-])\left(2.38\,\times 10^{-3}\,\left[\frac{slug}{ft^3}\right]\right)(23\,[ft^2])V^3 + (0.02\,[-])(3350\,[lb])V$$

This equation can be solved using MATLAB to obtain:

$$\boxed{V = 200.492\,\left[\frac{ft}{s}\right] \times \frac{3600\,s \cdot mi}{5280\,ft \cdot hr} = 136.7\,\left[\frac{mi}{hr}\right]}$$

But wait, there's more! We need to double check the restraint imposed by the transmission. The car can only go as fast as the redline that the highest gear allows. Note that for every 0.67 turns of the engine in sixth gear, the driveshaft will turn one time. From the final drive ratio, the wheels will turn one time for every 3.81 turns of the driveshaft. Consider the following set of gears representing the engine in gear six connected to the final drive:



$$\omega_{gear} = \frac{\omega_{engine}}{0.67}$$

$$\omega_{FD} = \frac{\omega_{gear}}{3.81} = \frac{\omega_{engine}}{0.67\,\times\,3.81}$$

We will use the redline of sixth gear coupled with the wheel radius and gear ratios to determine the allowable top speed as such:

$$V = \frac{\omega_{engine}}{GR_6 \times FDR} \times r_{wheel}$$

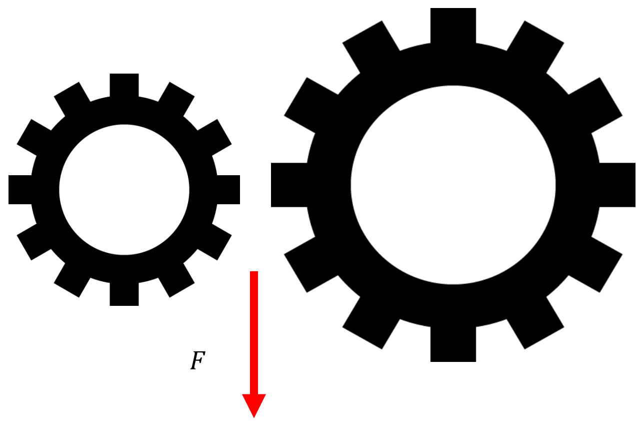
$$V = \frac{(6250\,[rpm])\left(\frac{2\pi\,min \cdot rad}{60\,rev \cdot s}\right)}{(0.67\,[-])(3.81\,[-])} \times \frac{26\,[in]}{2 \times 12\,\left[\frac{in}{ft}\right]}$$

$$\boxed{V = 277.8 \left[\frac{ft}{s}\right] = 189.4 \left[\frac{mi}{hr}\right]}$$

We thus conclude that the top speed of the Altima must be approximately 136.7 miles per hour and is limited by the external resistive forces on the vehicle.

<u>Part 2.)</u> Calculate the theoretical 0-60 mph time of the Altima.

To start, let's examine a set of gears.

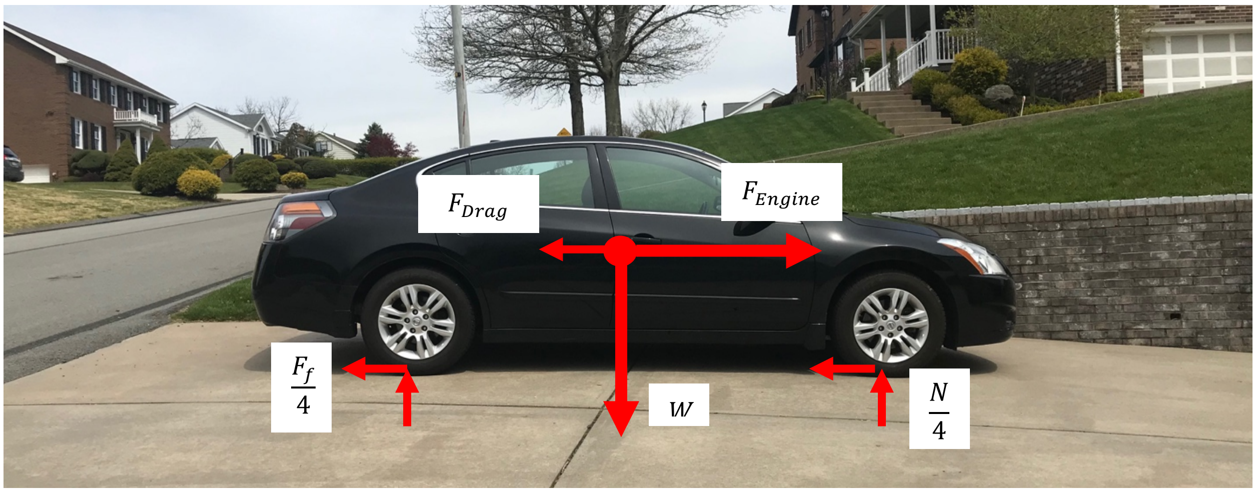


$$T_1 = r_1 F$$

$$T_2 = r_2 F = \frac{r_2}{r_1} T_1 = GR \cdot T_1$$

Note that Newton's Second Law holds that the forces exerted on each gear are equal and opposite each other. Thus, the torque transmitted from one gear to the next must scale by a factor of the radius of the output gear to that of the input gear, or in other words the <u>gear ratio</u>. To get the torque transmitted to the wheels, simply multiply the engine torque by the gear ratio, final drive axle ratio, and efficiency. The force is the wheel torque divided by the wheel radius.

The Free-Body Diagram is the same as the last problem:



The kinetic diagram now includes acceleration:

$$\Sigma F_{ext} = ma$$

$$\frac{T_{wheel}}{r_{wheel}} - F_{AR} - F_f = m\frac{dV}{dt}$$

$$\frac{T_{1,avg}}{r_{wheel}} - \frac{1}{2}c_d\rho AV^2 - \mu_r F_n = ma$$

Note the following relationships that will be important to our analysis:

$$T_{wheel} = 0.85T_{engine}(GR)(FDR)$$

$$V = \frac{\omega_{engine}}{(GR)(FDR)}$$

For first gear, we obtain the following values:

$$V_{maximum} = \frac{(6250\ [rpm])\left(\frac{2\pi\ min \cdot rad}{60\ rev \cdot s}\right)}{(3.5)(3.81)} = 49.081\ \left[\frac{ft}{s}\right] = 33.464\ \left[\frac{mi}{hr}\right]$$

$$T_{1,avg} = \frac{0.85}{49.081\frac{ft}{s}}\int_0^{49.081} T_1(V)dV$$

$$T_{1,avg} = 1632.048\ [ft \cdot lb]$$

$$F_{AR} = \frac{1}{60\ [mph]\left(\frac{5280}{3600}\frac{ft \cdot h}{mi \cdot s}\right)}\int_0^{60\ [mph]\left(\frac{5280 ft \cdot h}{3600 mi \cdot s}\right)} \frac{1}{2}c_d\rho AV^2 dV$$

$$F_{Ar} = 21.901\ [lb]$$

$$F_f = \mu_r F_n = (0.02\ [-])(3350\ [lb]) = 67\ [lb]$$

And therefore, we can solve for average acceleration through first gear:

$$\frac{T_{1,avg}}{r_{wheel}} - \frac{1}{2} c_d \rho A V^2 - \mu_r F_n = m\vec{a}$$

$$\frac{1632.048\ [ft \cdot lb]}{\frac{13}{12}[ft]} - 21.904[lb] - 67[lb] = \frac{3350\ [lb]}{32.174\ \left[\frac{ft}{s^2}\right]} a$$

$$a = 13.615 \left[\frac{ft}{s^2}\right]$$

We can now solve for the time it takes to reach the aforementioned velocity:

$$a = \frac{\Delta V}{t}$$

$$t = \frac{49.081 \left[\frac{ft}{s}\right]}{13.615 \left[\frac{ft}{s^2}\right]} = 3.605\ [s]$$

We will now repeat this process for second gear to reach 60 [mph].

$$V_{maximum} = \frac{(6250\ [rpm])(\frac{2\pi\ min \cdot rad}{60\ rev \cdot s})}{(1.95)(3.81)} = 88.094\ \left[\frac{ft}{s}\right] \approx 60\ [mph]$$

$$T_{2,avg} = \frac{0.85}{88.094\ \left[\frac{ft}{s}\right] - 49.081 \left[\frac{ft}{s}\right]} \int_{49.081}^{88.094} T_2(V) dV$$

$$T_{2,avg} = 1099.73\ [ft \cdot lb]$$

$$\frac{T_{2,avg}}{r_{wheel}} - \frac{1}{2} c_d \rho A V^2 - \mu_r F_n = ma$$

$$\frac{1099.73\ [ft \cdot lb]}{\frac{13}{12}[ft]} - 21.904[lb] - 67[lb] = \frac{3350\ [lb]}{32.174\ \left[\frac{ft}{s^2}\right]} a$$

$$a = 8.896\ \left[\frac{ft}{s^2}\right]$$

$$a = \frac{\Delta V}{t}$$

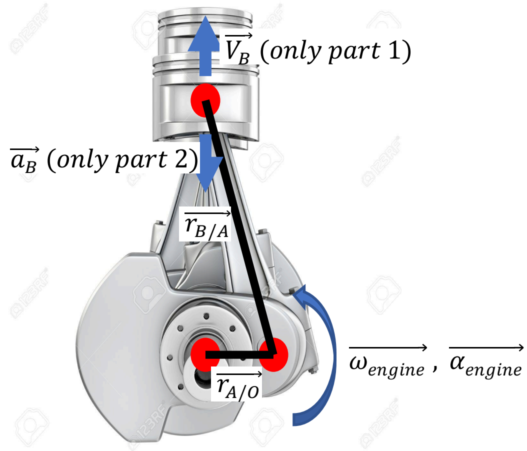
$$t = \frac{88.094\ \left[\frac{ft}{s}\right] - 49.081\left[\frac{ft}{s}\right]}{8.896\left[\frac{ft}{s^2}\right]} = 4.385\ [s]$$

We finally arrive at our solution for the 0-60 time!

$$\boxed{t = 3.605\ [s] + 4.385\ [s] = 7.990\ [s]}$$

Part 3.)

To envision what is happening in the engine, we'll look at the diagram of the velocities and accelerations of the piston:



The angular velocity of the crankshaft is given by:

$$\overrightarrow{\omega_{A/O}} = (6250\ [rpm])\left(\frac{2\pi\ min \cdot rad}{60\ rev \cdot s}\right)\hat{k}$$

The radius from the center of the crankshaft to the connecting rod at the position shown is given by:

$$\overrightarrow{r_{A/O}} = \left(\frac{2}{12}\ [ft]\right)\hat{\imath}$$

The velocity of the bottom of the connecting rod at the position shown is thus:

$$\overrightarrow{V_A} = \ (6250\ [rpm])\left(\frac{2\pi\ min \cdot rad}{60\ rev \cdot s}\right)\hat{k}\ \times \left(\frac{2}{12}\ [ft]\right)\hat{\imath} = 109.08\ \left[\frac{ft}{s}\right]\hat{\jmath}$$

By our relative velocity and geometric relationships:

$$\overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{B/A}}$$

$$V_B\hat{\jmath} = 109.08\ \left[\frac{ft}{s}\right]\hat{\jmath} + \overrightarrow{\omega_{B/A}} \times \overrightarrow{r_{B/A}}$$

$$\overrightarrow{r_{B/A}} = \left(-\frac{2}{12}[ft]\right)\hat{\imath} + \left(\frac{\sqrt{6.495^2 - 2^2}}{12}[ft]\right)\hat{\jmath} = \left(-\frac{2}{12}[ft]\right)\hat{\imath} + \left(\frac{6.179}{12}[ft]\right)\hat{\jmath}$$

After multiplying out the cross product, we develop the following set of equations that can be solved simultaneously:

$$\hat{\imath}\ Direction\colon\quad 0 = 0 - \frac{6.179}{12}\omega_{B/A}$$

$$\hat{\jmath}\ Direction\colon\quad V_B = 109.08\left[\frac{ft}{s}\right] - \frac{2}{12}\omega_{B/A}$$

Solving yields:

$$\omega_{B/A} = 0\left[\frac{rad}{s}\right]$$

$$\boxed{|V_B| = 109.08\left[\frac{ft}{s}\right] = 74.373\left[\frac{mi}{hr}\right]}$$

Now we will examine the acceleration of the pistons. From Part 2.) We must first perform a velocity analysis similar to the calculations above.

The velocity of the bottom of the connecting rod at the new position is:

$$\overrightarrow{V_A} = (6250\,[rpm])\left(\frac{2\pi\ min\cdot rad}{60\ rev\cdot s}\right)\hat{k}\times\left(\frac{2}{12}\,[ft]\right)\hat{\jmath} = -109.08\left[\frac{ft}{s}\right]\hat{\imath}$$

By our relative velocity and geometric relationships:

$$\overrightarrow{V_B} = \overrightarrow{V_A} + \overrightarrow{V_{B/A}}$$

$$V_B\hat{\jmath} = -109.08\left[\frac{ft}{s}\right]\hat{\imath} + \overrightarrow{\omega_{B/A}}\times\overrightarrow{r_{B/A}}$$

$$\overrightarrow{r_{B/A}} = \left(\frac{6.495}{12}[ft]\right)\hat{\jmath}$$

After multiplying out the cross product, we develop the following set of equations that can be solved simultaneously. Note that the velocity of point B can only be in the vertical direction.

$$\hat{\imath}\ Direction\colon\quad 0 = -109.08 - \frac{6.495}{12}\omega_{B/A}$$

$$\hat{\jmath}\ Direction\colon\quad V_B = 0$$

Solving yields:

$$\omega_{B/A} = -201.53\left[\frac{rad}{s}\right]$$

$$V_B = 0\left[\frac{ft}{s}\right]$$

We can now focus on acceleration and calculate the average angular acceleration of the crankshaft in first gear as we reach the redline from rest:

$$\overrightarrow{\alpha_{A/O}} = \frac{(6250\ [rpm])\left(\frac{2\pi\ min \cdot rad}{60\ rev \cdot s}\right) - 0\left[\frac{rad}{s}\right]}{3.605\ [s]}\hat{k} = 181.55\ \left[\frac{rad}{s^2}\right]\hat{k}$$

The acceleration of point A is given by:

$$\overrightarrow{a_A} = [\overrightarrow{\omega_{A/O}} \times (\overrightarrow{\omega_{A/O}} \times \overrightarrow{r_{A/O}})][\hat{k} \times (\hat{k} \times \hat{j})] + [\overrightarrow{\alpha_{A/O}} \times \overrightarrow{r_{A/O}}][\hat{k} \times \hat{j}]$$

$$\overrightarrow{a_A} = -\alpha_{A/O}\, r_{A/O}\hat{\imath} - r_{A/O}\omega_{A/O}{}^2\hat{\jmath}$$

$$\overrightarrow{a_A} = -30.258\left[\frac{ft}{s^2}\right]\hat{\imath} - 71394.71\left[\frac{ft}{s^2}\right]\hat{\jmath}$$

Now, using the relationships for relative acceleration between point A and B,

$$\overrightarrow{a_B} = \overrightarrow{a_A} + \overrightarrow{a_{B/A}}$$

$$\overrightarrow{a_B} = \overrightarrow{a_A} + [\overrightarrow{\omega_{B/A}} \times (\overrightarrow{\omega_{B/A}} \times \overrightarrow{r_{B/A}})][\hat{k} \times (\hat{k} \times \hat{j})] + [\overrightarrow{\alpha_{B/A}} \times \overrightarrow{r_{B/A}}][\hat{k} \times \hat{j}]$$

$$\overrightarrow{a_B} = \overrightarrow{a_A} - \alpha_{B/A}\, r_{B/A}\hat{\imath} - r_{B/A}\omega_{B/A}{}^2\hat{\jmath}$$

This produces a set of equations for each component direction. Note that the linear acceleration of point B can only be in the vertical direction.

$$\hat{\imath}\ Direction\colon\quad 0 = -30.258 - \frac{6.495}{12}\alpha_{B/A}$$

$$\hat{\jmath}\ Direction\colon\quad a_B = -71394.71 - \frac{6.495}{12}\omega_{B/A}{}^2$$

Solving simultaneously yields

$$\alpha_{B/A} = -55.904\ \left[\frac{rad}{s^2}\right]$$

$$\boxed{\overrightarrow{a_B} = -93377.22\ \left[\frac{ft}{s^2}\right]\hat{\jmath}}$$

That’s a lot of feet per second squared, almost boggles the mind.

I know this report was a little long, but I was excited to do this analysis because I am very passionate about my car, and I learned some things that I didn't know before. **Thanks for bearing with me, and thanks for a great semester Dr. Kerzmann!**

Links:

[1] https://www.automobile-catalog.com/curve/2011/2294555/nissan_altima_coupe_2_5_s_cvt.html
[2] https://www.iseecars.com/car/2007-nissan-altima-specs
[3] https://ecomodder.com/wiki/Vehicle_Coefficient_of_Drag_List
[4] https://www.andysautosport.com/products/brian_crower__BC6218.html
[5] https://www.engineeringtoolbox.com/rolling-friction-resistance-d_1303.html