Design Project #1

DESIGN OF A WOODEN PILE-AND-PLANK RETAINING WALL

ENGR 0145
STATICS AND MECHANICS OF MATERIALS 2

Dr. William Slaughter

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Lucas Bechtold
Frank Czura
John Maglosky
Sahana Mallela

Abstract

This report details the research, calculations, and code that was performed by the group in order to solve the problem within the parameters of the design project itself. This included determining the lowest cost possible to construct an adequately performing retaining wall 80 feet wide and 5 feet tall using pressure-treated standard structural timber

The problem was approached by assuming the planks to be simply supported beams bridging the gap between piles, enabling the planks to be modeled with an even force distribution to find the minimum section modulus needed for the planks. Similarly, the piles were assumed to be cantilevered beams with an asymmetrical force distribution, allowing the calculation of the minimum section modulus for the piles. The equations gathered from the analysis of the planks and piles were then inputted to a MATLAB script where the cheapest possible combination was determined.

Introduction

Typically used in marine applications, pile-and-plank retaining walls have become increasingly popular in residential and public areas, offering a variety of design options. Their design generally consists of horizontal planks supported by vertical piles and are used to create a terrace in sloped terrain, helping mediate erosion caused by storms and allowing for the topography of an area to be modified. Due to the nature of these walls, they are faced with high loads due to the backfill of soil and must be designed for such.

Using pressure-treated Standard Structural Timber, an 80-foot-long, 5-foot-tall terrace must be constructed with an allowable flexural stress of 1200 psi. The load faced by the wall acts laterally against it and begins at 500 lb/ft2 at the base, which then linearly decreases to 100 lb/ft2 at the top of the wall. The piles should extend 5 feet underground, be evenly spaced with piles at either end of the wall and have a square cross section. The planks should all be the same size and have the same length as the spacing between the piles, such that each plank is supported by a pile at each end. The wall should be designed to minimize cost, given a \$14/ft^3 cost for the timber, available in 8, 10, and 12-foot lengths, accompanied with a \$40 per pile cost for the concrete footing necessary for support.

We were able to ensure that we came up with the strongest and cheapest beam through the results we were able obtain using hand and iterative calculations through MATLAB. Without these clear measurable results, we would be unable to be as certain about the success of our design. We were able to obtain the minimum section modulus of our planks and piles of the retaining wall by hand, in addition to determining which plank sizes were possible by analyzing the divisibility of the plank lengths and the total timber sizes of 8, 10, and 12 ft. Then, using MATLAB, we were able to acquire the costs of each combination of possible planks and pile spacings, the results of which were then checked by hand.

Analysis & Design

In order to design the most cost-effective retaining wall within the constraints specified above, our group decided that an iterative approach using MATLAB would be the best way to ensure that we fell within allowable stress limits and achieved the lowest overall cost. Before we could begin to code, however, it was necessary to perform symbolic calculations so that we could explore the entire solution space.

We shall begin with the analysis of the planks, which are the horizontal beams in between the vertical piles. We made several assumptions regarding the planks, including:

1. The highest pressure, 500 $\frac{lb}{ft^2}$, is uniformly distributed across the bottom plank. Thus, we conduct a stress analysis of the bottom plank as the "worst-case". Designing for the highest pressure on a plank ensures that all planks are strong enough to support the prescribed loads.

2. The planks can effectively be modeled as simply supported beams with the dimensions of the piles considered negligible. If the pile dimensions were significant, they would in effect reduce the length of a plank under load and thus reduce the maximum bending moment experienced by the plank.

3. All planks in the wall are to be the same dimensions, and the span of each plank is the same (as a consequence of equal pile spacing).

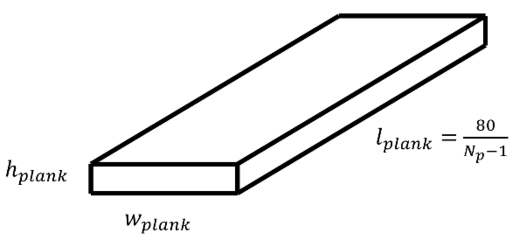


Fig. 1 The dimensions of a plank. If there are Np piles, there will be Np-1 planks between them.

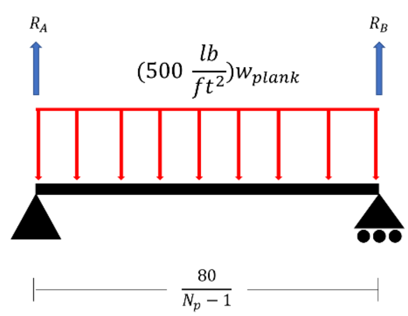


Fig. 2 FBD of the simply supported plank. The distributed load across the length takes into account the pressure acting along the width of the plank to be determined.

From the equations for equilibrium, expressions can be constructed for the reactionary forces at A and B.

$$\Sigma M_A = 0\ lb * ft$$

$$-500w_{plank}l_{plank}\left(\frac{l_{plank}}{2}\right) + R_B l_{plank} = 0$$

$$-500w_{plank}\left(\frac{40}{N_P - 1}\right)\left(\frac{80}{N_P - 1}\right) + R_B\left(\frac{80}{N_P - 1}\right) = 0$$

$$R_B = 500w_{plank}\left(\frac{40}{N_P - 1}\right) = \frac{20000w_{plank}}{N_p - 1}$$

$$R_A = 500w_{plank}\left(\frac{40}{N_P - 1}\right) = \frac{20000w_{plank}}{N_p - 1} \qquad (by\ symmetry)$$

From this, expressions for the shear force and bending moment at any point in the beam can be constructed according to the free-body diagram below.

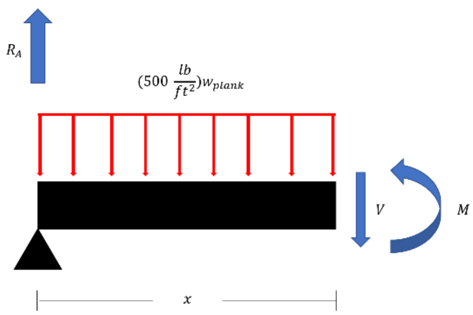


Fig. 3 Reactions existing at an arbitrary cross-section of a given plank.

$$V(x) = \frac{20000w_{plank}}{N_P - 1} - 500w_{plank}x$$

No reactionary moments exist at the pins, so the integration constant for the shear force equation is zero.

$$M(x) = \int V(x)$$

$$M(x) = \frac{20000w_{plank}}{N_P - 1}x - 250w_{plank}x^2$$

The maximum shear force for this beam exists at either of the simple supports, and the maximum bending moment occurs halfway along the beam. Thus, making the appropriate substitutions,

$$V_{max} = \frac{20000w_{plank}}{N_P - 1}$$

$$M_{max} = \frac{20000w_{plank}}{N_P - 1}\left(\frac{40}{N_P - 1}\right) - 250w_{plank}\left(\frac{40}{N_P - 1}\right)^2$$

The maximum flexural stress in a beam is equal to the maximum bending moment experienced by the beam divided by the section modulus of a beam. Using the expression for the maximum bending moment, we can develop an expression for the minimum section modulus required for a given pile spacing and plank width:

$$\sigma_{max} = \frac{M_{max}}{S_{min}}$$

$$S_{min} = (\frac{20000 w_{plank}}{N_P - 1}\left(\frac{40}{N_P - 1}\right) - 250 w_{plank}\left(\frac{40}{N_P - 1}\right)^2)/172800\frac{lb}{ft^2}$$

Now we shall analyze the piles that are supporting the planks. We made several assumptions regarding the piles, including:

1. The piles are all evenly spaced and square in cross-section.

2. The piles act as cantilevered beams with a pressure distribution linearly decreasing from 500 $\frac{lb}{ft^2}$ at the base of the pile to 100 $\frac{lb}{ft^2}$ at the top of the pile.

3. The maximum bending moment at any cross-section of the pile occurs at the base where the reactionary moment exists. This assumption will be verified following further analysis.

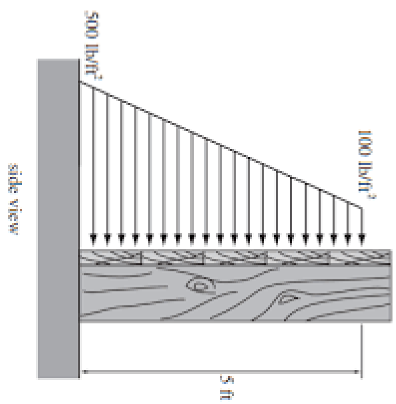


Fig. 4 Rotation of the pile in space to represent a cantilevered beam. Note the reactionary force and moment existing at the base and the equivalent point load.

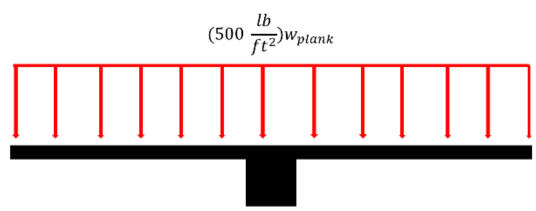


Fig. 5 Each beam functions as a simple support for two different planks, carrying half the load of each plank by symmetry. Thus, the force experienced by one pile is equivalent to the force acting along one complete pile spacing.

The pressure distribution for the piles can be modeled as:

$$P(x) = 500 - 80x \qquad 0 < x < 5$$

Therefore, the equivalent point load along a length of pile can be expressed as the integral of the pressure distribution (yields force per length of plank) times the pile spacing:

$$L_{eq} = \int_0^5 (500 - 80x)dx\left(\frac{80}{N_P - 1}\right)$$

$$L_{eq} = \frac{120000}{N_P - 1}$$

The point load acts at the centroid of the pressure distribution, which can be obtained by calculating a weighted average of the rectangular and triangular components of the overall pressure distribution and exists at

$$x_c = 1.944\bar{4}\ ft$$

The equivalent point load and centroid of the distribution allows us to calculate the reactionary moment at the base of the beam, which is taken to be the maximum bending moment experienced by the beam:

$$M_R = M_{max} = L_{eq} x_c = \frac{120000}{N_P - 1}(1.944\bar{4})$$

$$M_R = \frac{233333.3\bar{3}}{N_P - 1}$$

Using the expression given above for the minimum section modulus, the minimum section modulus of a pile for a given pile spacing is given by:

$$S_{min} = \frac{1.35030864}{N_P - 1}$$

Now that we have developed expressions constraining the possible combinations of piles and planks with regard to strength, we must consider the total cost of all possible combinations of standard structural timbers, which are available for purchase in 8, 10, or 12-foot lengths. Regarding the pile lengths, the wall must be at least five feet high, which implies that five feet must also extend below the ground. Thus, if a plank width is chosen that can be multiplied by an integer to give five feet, a ten-foot pile length is sufficient. However, if a plank width is chosen that does not evenly go into five feet, the number of planks must be rounded up so that the vertical height of the wall is greater than five feet. In this case, a 12-foot length of timber must be purchased for each pile and cut to the appropriate size. The timber costs $14 per cubic foot and has a fixed cost of $40 per pile. The cost of the piles is given by the following:

$$C_{pile} = (14)(10)w_{pile}^2 N_P + 40N_P \quad if\ \frac{5}{w_{plank}}\ is\ an\ integer$$

$$C_{pile} = (14)(12)w_{pile}^2 N_P + 40N_P \quad if\ \frac{5}{w_{plank}}\ is\ not\ an\ integer$$

Regarding the planks, the spacing between the piles is equal to the length of the planks. The minimum number of piles is eight, because any less would result in a pile spacing greater than the longest length of timber available for purchase. We are limited to a maximum of 41 piles.

The pile spacing thus ranges from two feet to twelve feet. In this range, there are certain values for pile spacing that would allow us to use a full length of timber with no waste. These are summarized in the table below:

Pile Spacing (ft)	**Possible # of Planks**	**Timber Length Used (ft)**
2	4	8
2.4	5	12
2.5	4	10
8/3	3	8
3	4	12
10/3	3	10
4	2	8
5	2	10
6	2	12
8	1	8
10	1	10
12	1	12

For pile spacings between those given in the table, the best option for a given pile spacing is the same as the next-longest pile spacing. For instance, for a pile spacing of 2.75 feet, it is better to buy a 12-foot length, produce four planks, and waste one foot as opposed to buying an 8-foot length, producing 2 planks, and wasting 2.5 feet.

For any given pile spacing, the cost of the planks is given by:

$$\#\ of\ rows = \frac{5\ ft}{w_{plank}}\ \ rounded\ up\ if\ not\ whole\ \#$$

$$\#\ of\ planks = (\#\ of\ rows)(N_p - 1)$$

$$\#\ of\ timbers = \frac{\#\ of\ planks}{Possible\ \#\ of\ planks\ per\ timber}\ \ rounded\ up\ if\ not\ whole\ \#$$

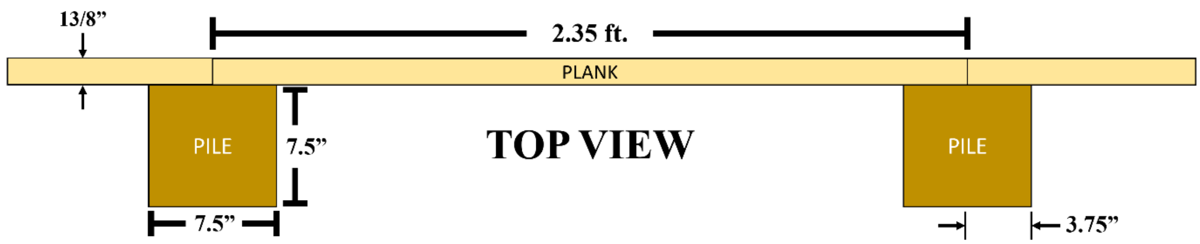
$$C_{planks} = (\#\ of\ timbers)(plank\ cross\ section)(timber\ length)(14)$$

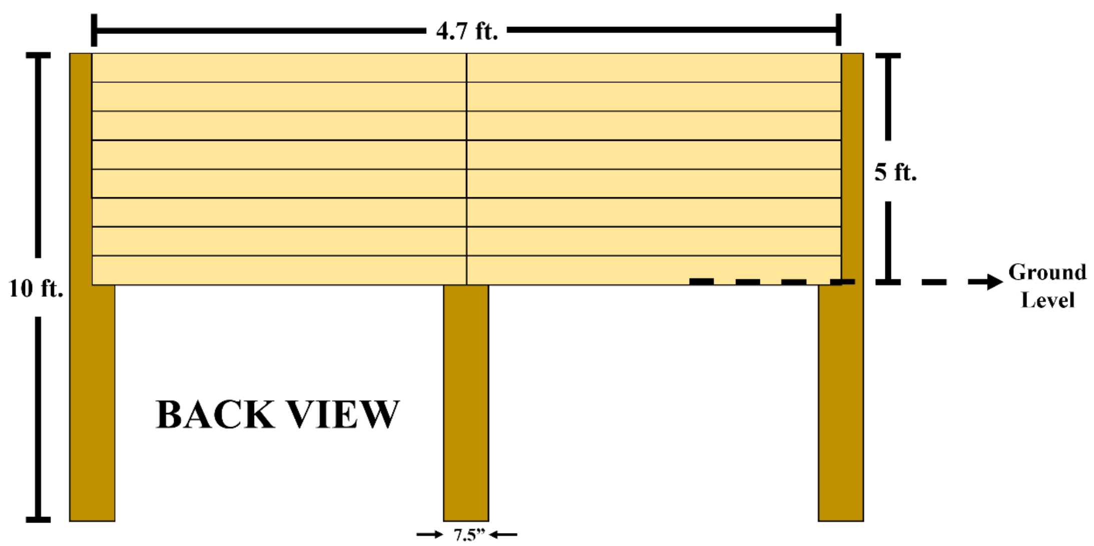
With these relationships established, our team developed a MATLAB script for an iterative approach to the problem of designing the cheapest safe wall. The script first converts the data from Table A-15 into base units of feet. Section modulus values used for each timber were manually adjusted, as the orientation of the beam for the section modulus values in Table A-15 did not match the orientation for application in our wall.

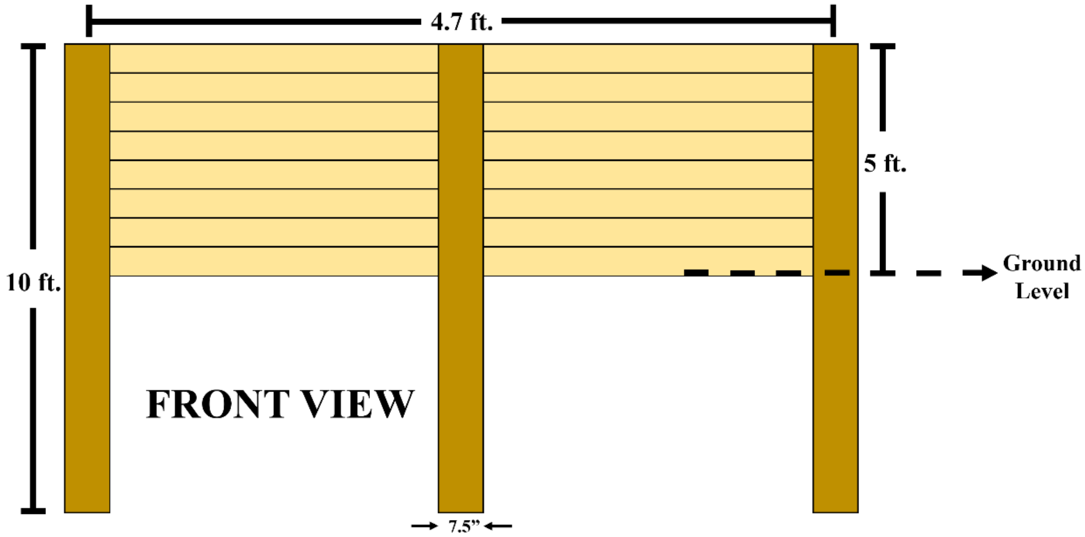
The script uses nested loops along with conditional statements and the formulas developed for minimum section moduli of the planks and piles to test every combination of pile spacing, plank dimensions, and pile dimensions possible for the wall. Only those with section moduli greater than the minimum were added to a possibility's matrix. Then, using further iteration and conditional statements, the cost of the piles and planks was calculated using the above formulas for each combination that passed the section modulus test. The script then compared all costs and identified the cheapest possible combination of piles and planks. Running the script returns the following results:

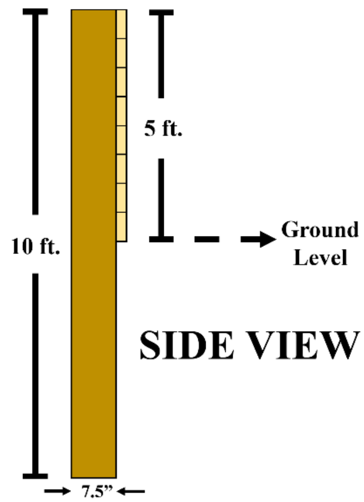
- Pile spacing: 2.3529 (80/34) feet
- Number of piles: 35
- Pile dimensions: 8 in x 8 in x 10 ft (7.5 in x 7.5 in x 10 ft dressed size)
- Plank dimensions: 2 in x 8 in x 8 ft (13/8 in x 7.5 in x 8 ft dressed size)
- Number of planks: 272
- Cost: $4096.10

These dimensions correspond to the following design diagrams (not to scale):









Results

After analyzing all possible retaining walls, our MATLAB script identified the aforementioned characteristics of the most cost-effective, safe retaining wall. Using the equations developed above, we manually verified that the section modulus values for the piles and planks exceeded the minimum section modulus values, given pile spacing and plank width:

$$\boxed{S_{min,plank} = 0.001252\ ft^3}$$

$$S_{2x8} = 0.001910\ ft^3 \ > \ S_{min,plank}$$

$$\boxed{S_{min,pile} = 0.03971\ ft^3}$$

$$S_{12x12} = 0.04069\ ft^3\ >\ S_{min,pile}$$

Additionally, to verify our assumption that the maximum bending moment in the pile occurs at the base of the pile, we expressed the bending moment at any point in the beam in terms of x as:

$$M(x) = M_R + (4000 - 320x)(x)\frac{2000x^2 - \frac{640x^3}{3}}{4000x - 320x^2}$$

The first term in the above equation represents the average distributed load along the pile for a given length x. The second term is the length of the pile, giving the total load on the pile for a length x. The final term represents the centroid of the distributed load across the arbitrary cross-section. Using MATLAB to differentiate this expression, we found that the only critical point is at x=25/4, which lies beyond the end of the beam. This expression is therefore strictly increasing over the length of the beam, and thus our assumption regarding the maximum bending moment is correct.

The following chart summarizes the overall cost of our beam:

Component	**Piles**	**Planks**
Dimensions	7.5 in x 7.5 in x 10 ft	13/8 in x 7.5 in x 12 ft
Volume Per Unit	3.90625 ft^3	1.015625 ft^3
Number Required	35	55
Unit Cost	$54.6875	$14.21875
Cost	$1914.0625	$782.03125

Therefore:

$$Final\ Cost = Cost_{piles} + Cost_{planks} + \$40(Number\ of\ Piles)$$

$$= \$1914.0625 + \$782.03215 + \$40(35)$$

$$\boxed{\rightarrow\ Final\ Cost = \$4096.10}$$

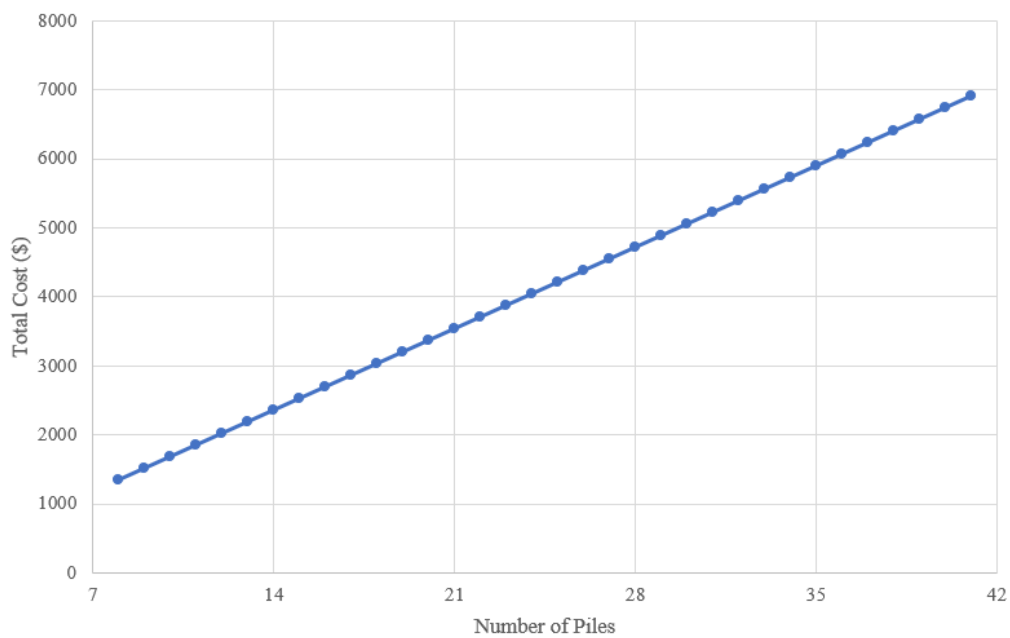
Now, to plot the cost of the retaining wall versus the number of piles used, we need to determine which equation of C_{pile} to use. From Table A-15,

$$w_{plank} = 7.5\ in \rightarrow \frac{5}{w_{plank}} = \frac{5}{7.5\ in * \frac{1\ ft}{12\ in}} = 8\ ft^{-1}$$

Because this value is an integer, the equation used to calculate C_{pile} is:

$$C_{pile} = (14)(10)w_{pile}^2 N_p + 40N_p \mid w_{pile} = 7.5\ in$$

Which when plotted from $8 \leq N_p \leq 41$ piles gives graph:



In summary, after careful iterative analysis of the stresses existing in the planks and piles, we determined that the most cost-effective beam has 35 eight-by-eight evenly spaced piles and eight rows of two-by-eight planks spanning the gaps. This combination ensures that the allowable stresses are not exceeded, and the cost is as low as possible.